

Chapter 5

Properties of Triangles

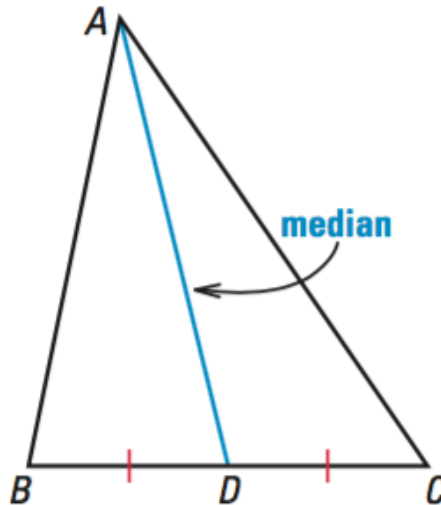
Section 3

Medians and Altitudes of a Triangle

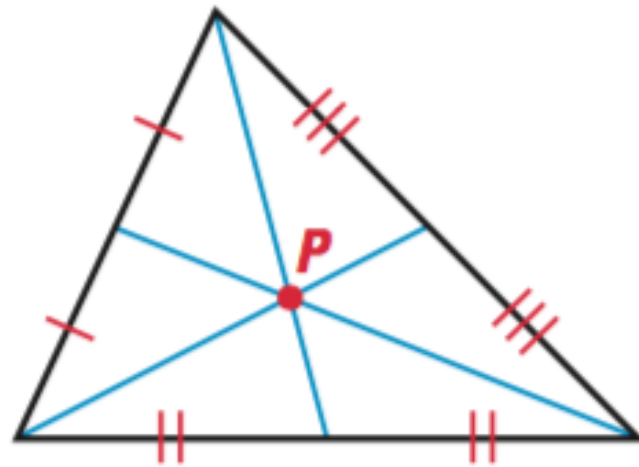
GOAL 1: Using Medians of a Triangle

In Lesson 5.2, you studied two special types of segments of a triangle: perpendicular bisectors of the sides and angle bisectors. In this lesson, you will study two other special types of segments of a triangle: medians and altitudes.

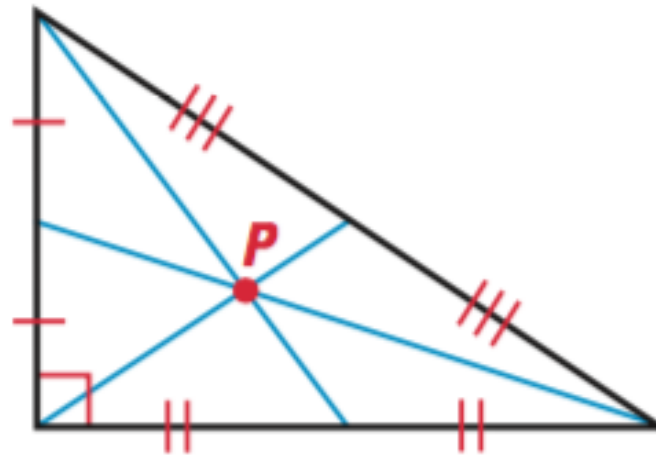
A _____ median _____ is a segment whose endpoints are a vertex of the triangle and the midpoint of the opposite side. For instance, in $\triangle ABC$ shown at the right, D is the midpoint of side BC . So, AD is a median of the triangle.



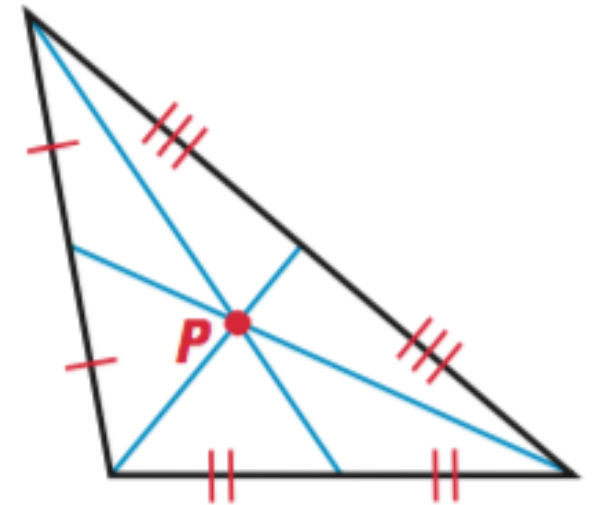
The three medians of a triangle are concurrent. The point of concurrency is called the _____centroid_____. The centroid, labeled P in the diagrams below, is always inside the triangle.



acute triangle



right triangle



obtuse triangle

THEOREM

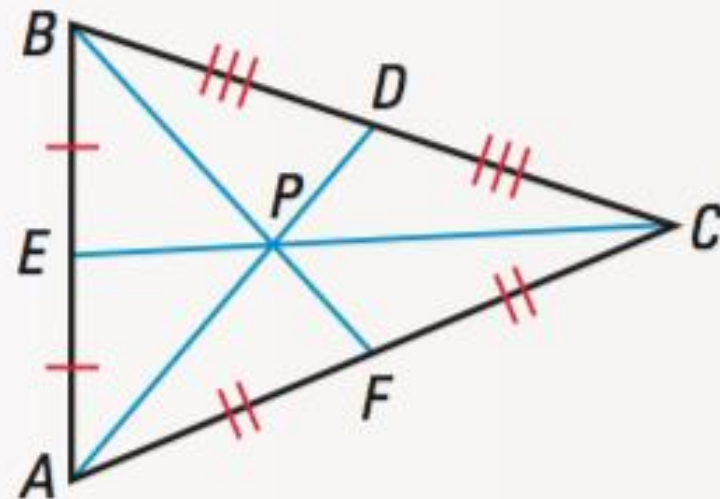
THEOREM 5.7 *Concurrency of Medians of a Triangle*

The medians of a triangle intersect at a point that is two thirds of the distance from each vertex to the midpoint of the opposite side.

If P is the centroid of $\triangle ABC$, then

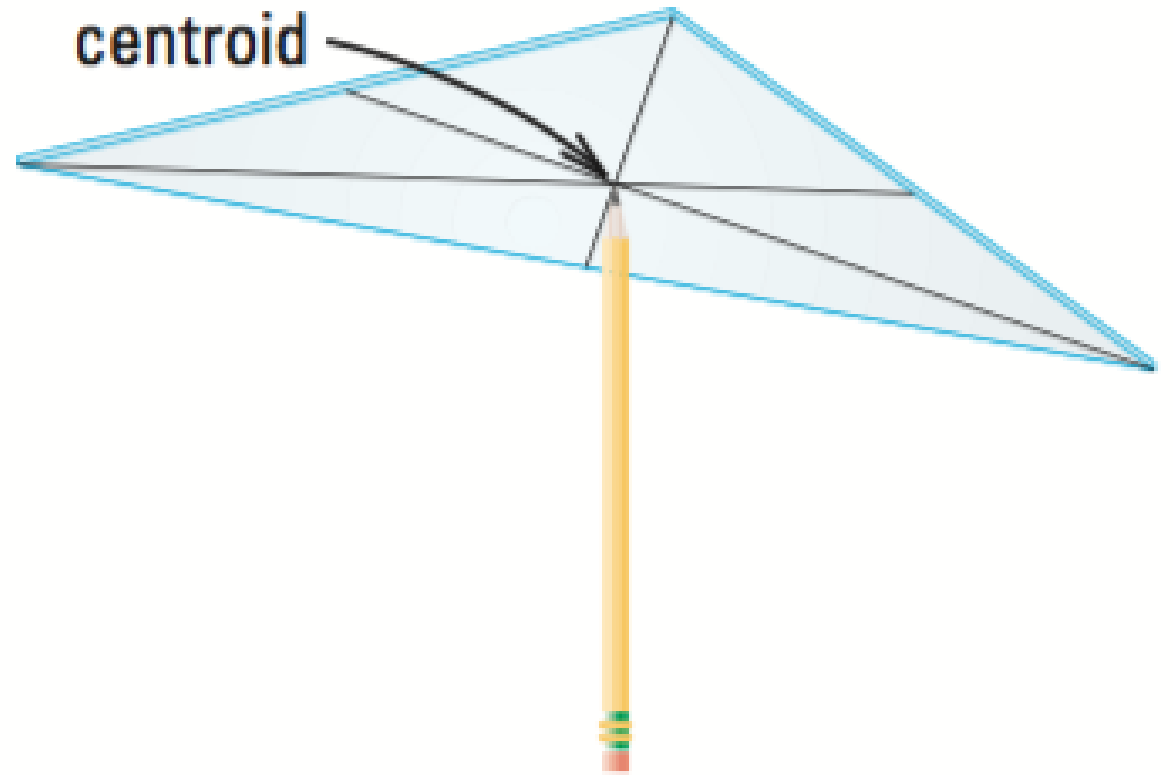
$$AP = \frac{2}{3}AD, BP = \frac{2}{3}BF, \text{ and } CP = \frac{2}{3}CE.$$

$$PD = \frac{1}{3}AD, PF = \frac{1}{3}BF, \text{ and } PE = \frac{1}{3}CE$$



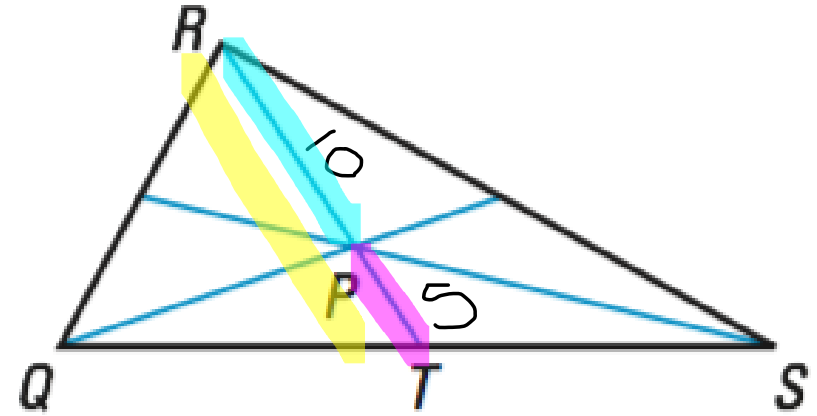
The centroid of a triangle can be used as its balancing point.

A triangular model of uniform thickness and density will balance at the centroid of the triangle. For instance, in the diagram shown at the right, the triangular model will balance if the tip of a pencil is placed at its centroid.



Example 1: Using the Centroid of a Triangle

P is the centroid of $\triangle QRS$ shown below and $PT = 5$. Find RT and RP .



$$RP \rightarrow 5 \times 2 = 10$$

$$RT \rightarrow 10 + 5 = 15$$

Example 2: Finding the Centroid of a Triangle

Find the coordinates of the centroid of $\triangle JKL$.

$$KN = 6$$

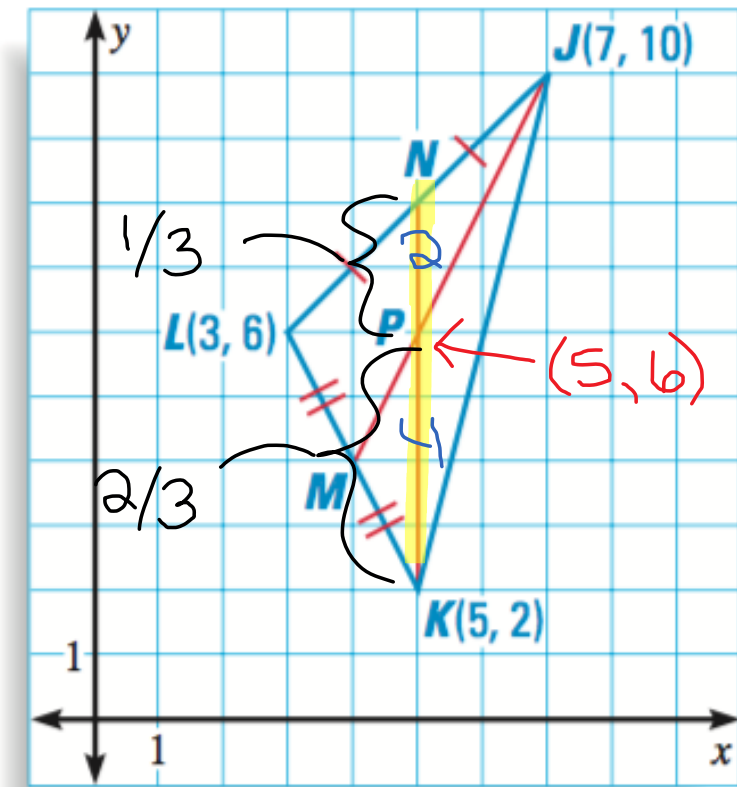
$$6/3 = 2 \rightarrow PN = 2$$

$$2 \times 2 = 4 \rightarrow PK = 4$$

Given whole:

whole / 3 = smaller part

smaller part \times 2 = bigger part



GOAL 2: Using Altitudes of a Triangle

An _____ altitude _____ is the perpendicular segment from a vertex to the opposite side or to the line that contains the opposite side. An altitude can lie inside, on, or outside the triangle.

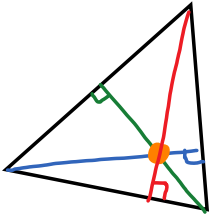
Every triangle has three altitudes. The lines containing the altitudes are concurrent and intersect at a point called the _____ orthocenter _____.

Example 3: Drawing Altitudes and Orthocenters

Where is the orthocenter located in each type of triangle?

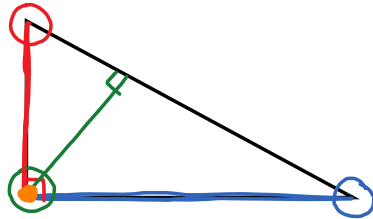
a) Acute

↳ inside



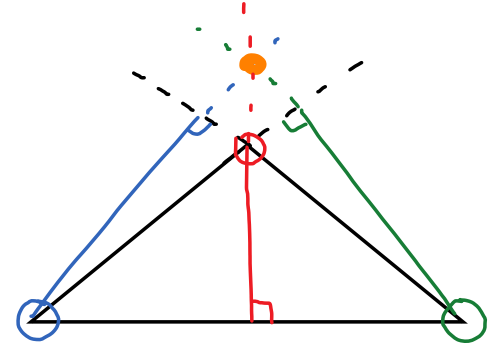
b) Right

↳ on



c) Obtuse

↳ outside

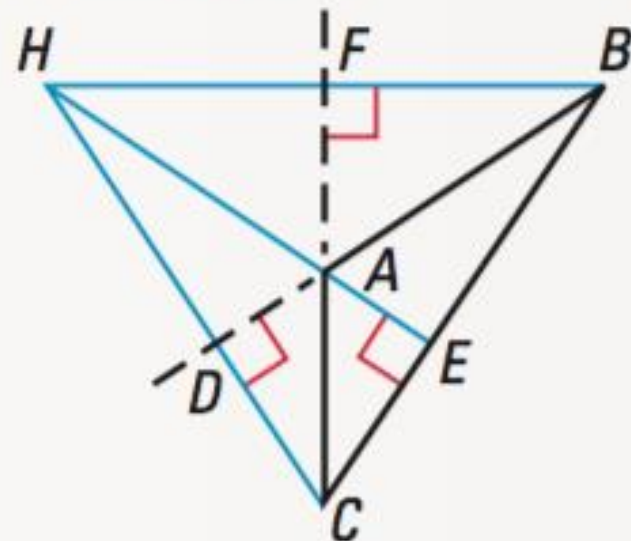


THEOREM

THEOREM 5.8 *Concurrency of Altitudes of a Triangle*

The lines containing the altitudes of a triangle are concurrent.

If \overline{AE} , \overline{BF} , and \overline{CD} are the altitudes of $\triangle ABC$, then the lines \overleftrightarrow{AE} , \overleftrightarrow{BF} , and \overleftrightarrow{CD} intersect at some point H .



EXIT SLIP